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Photon-added squeezed vacuum (one-photon) state as an even (odd) nonlinear coherent state

Nai-le Liu†‡, Zhi-hu Sun‡ and Hong-yi Fan‡

† CCAST(World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China
‡ Department of Material Science and Engineering, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China§

E-mail: liunai@mail.ustc.edu.cn

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Abstract. We show that the photon-added squeezed vacuum $a^{\dagger m}S(z)|0\rangle$ and the photon-added squeezed one-photon state $a^{\dagger m}S(z)|1\rangle$, where m is a non-negative integer, may be regarded as even and odd nonlinear coherent states, respectively. To achieve this, we derive an operator-valued function $f(N, m)$ of the number operator $N = a^\dagger a$ such that $a^{\dagger m}S(z)|i\rangle$ $(i = 0, 1)$ are eigenstates of $f(N, m)a^2$. Based on this, and using the unified method developed by Shanta *et al*, we find that $a^{\dagger m}S(z)|i\rangle$ can be equivalently expressed in exponential forms. The eigenstates of $f(N, -m)a^2$ are also constructed and their nonclassical features are studied in detail.

1. Introduction

Recently *nonlinear coherent states* (NLCSs) have attracted attention [1, 2]. These states are defined as the right-hand eigenstates of $f(N)a$, where $f(N)$ is a nonlinear operator-valued function of the number operator $N = a^\dagger a$ and $a(a^\dagger)$ is the boson annihilation (creation) operator. They may be regarded as a generalization of the q-deformed coherent states [3]. One special class of NLCSs could be generated as stationary states of the centre-of-mass motion of a laser-driven trapped ion far from the Lamb–Dicke limit [1]. The notion of NLCSs has been generalized to the *even and odd NLCSs* [4], defined as the eigenstates of $f(N)a^2$, and to the real and imaginary nonlinear Schrödinger cat states [5]. Another class of states that have been a subject of increasing interest are *photon-added states* that are obtained by repeated application of photon creation operators on a given state. The earliest examples in the literature are the photon-added coherent states by Agarwal and Tara [6]. It was shown in [6] that photon-added states can be produced in the processes of the field–atom interaction in a cavity. Since then, the photon-added squeezed states [7–9], the photon-added thermal states [10, 11] and the photon-added even and odd coherent states [12], etc, have been introduced. Most of the theoretical studies concerning these states have focused on their generations and the possible occurrence of various nonclassical effects exhibited by them. More recently, an effective method of generating photon-added states in a travelling light beam by means of conditional measurements on a beam splitter was also proposed [13, 14].

In this paper we show that the photon-added squeezed vacuum (PASV) and the photon-added squeezed one-photon state (PASOPS) can be viewed as special even and odd NLCSs,

§ Mailing address.

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respectively. Our original stimulus was the paper of Sivakumar [15], in which the photon-added coherent states are proven to be special NLCSs. The PASV was first studied by Zhang and Fan [7] and then generalized by others to photon-added squeezed coherent states [8,9]. In view of the fact that the squeezed vacuum (the even Fock subspace)

$$S(z)|0\rangle = \left(\frac{1}{\cosh r}\right)^{1/2} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{2^n n!} (\mathrm{e}^{\mathrm{i}\theta} \tanh r)^n |2n\rangle \tag{1}$$

and the squeezed one-photon state (the odd Fock subspace)

$$S(z)|1\rangle = \left(\frac{1}{\cosh r}\right)^{3/2} \sum_{n=0}^{\infty} \frac{\sqrt{(2n+1)!}}{2^n n!} (\mathrm{e}^{\mathrm{i}\theta} \tanh r)^n |2n+1\rangle \tag{2}$$

respectively correspond, from a group theoretical point of view, to two unitary irreducible representations of $su(1, 1)$ Lie algebra: $k = \frac{1}{4}$ and $k = \frac{3}{4}$ (k is the Bargmann index that labels irreducible representations of $su(1, 1)$) [16], we should study the PASV and the PASOPS as a whole. In the above equations

$$S(z) = \exp\left(\frac{z}{2}a^{\dagger 2} - \frac{z^*}{2}a^2\right) \qquad z = r\mathrm{e}^{\mathrm{i}\theta} \tag{3}$$

is the single-mode squeezing operator [17].

2. The PASV (PASOPS) as an even (odd) NLCS

Let us suppose that the (un-normalized) PASV $a^{\dagger m} S(z)|0\rangle$ and PASOPS $a^{\dagger m} S(z)|1\rangle$ (for the normalization of PASV, see, e.g., [7]) satisfy a unified eigenvalue equation as follows:

$$f(N, m)a^2 a^{\dagger m} S(z)|i\rangle = \alpha a^{\dagger m} S(z)|i\rangle \qquad i = 0, 1 \tag{4}$$

wherein m is a non-negative integer, $f(N, m)$ is a function of N and m to be determined, α is a complex parameter which should be related to z. Inserting equations (1) and (2) into (4) and taking into account the well known relations for the action of boson operators onto the Fock states (e.g. [18]), we obtain the following equation:

$$f(m+2n, m) = \frac{2n+2}{(m+2n+2)(m+2n+1)} \qquad (n = 0, 1, \ldots) \tag{5}$$

$$f(m+2n+1, m) = \frac{2n+2}{(m+2n+2)(m+2n+3)} \qquad (n = 0, 1, \ldots) \tag{6}$$

$$\alpha = \mathrm{e}^{\mathrm{i}\theta} \tanh r. \tag{7}$$

Expressions (5) and (6) are summarized as

$$f(n, m) = \begin{cases} (n-m+2)/[(n+1)(n+2)] & \text{if} \quad n-m \text{ is even} \\ (n-m+1)/[(n+1)(n+2)] & \text{if} \quad n-m \text{ is odd.} \end{cases} \tag{8}$$

In what follows, we use the notation $[x]$ to denote the greatest integer less than or equal to x. Then $f(n, m)$ is rewritten in a compact form:

$$f(n, m) = \frac{2[\frac{n-m+2}{2}]}{(n+1)(n+2)}. \tag{9}$$

Therefore, the PASV and the PASOPS can be identified with the even and odd NLCSs, respectively. When $m = 0$, (4) reduces to the eigenvalue equations satisfied by the squeezed vacuum and squeezed one-photon state:

$$\left(\frac{1}{N+1}a^2\right) S(z)|0\rangle = \alpha S(z)|0\rangle \qquad \left(\frac{1}{N+2}a^2\right) S(z)|1\rangle = \alpha S(z)|1\rangle. \tag{10}$$

3. Exponential forms of the PASV and the PASOPS

It is also possible to express the PASV and the PASOPS in equivalent exponential-operator forms. This can be achieved by using the procedure by Shanta *et al* [19, 20]. Let $F = f(N, m)a^2$. By a direct observation we find that the states which are annihilated by F are $|0\rangle$, $|1\rangle$, $|m\rangle$ and $|m+1\rangle$. Accordingly the Fock space is clearly split into four sectors. The sector S_j ($j = 0, 1, m, m+1$) is built out of $|j\rangle$ by repeated applications of $F^\dagger$

$$\begin{aligned}
S_0 &= \left\{|2n\rangle, n = 0, 1, \ldots, \left[\frac{m-1}{2}\right]; m \geqslant 1\right\} \\
S_1 &= \left\{|2n+1\rangle, n = 0, 1, \ldots, \left[\frac{m-2}{2}\right]; m \geqslant 2\right\} \\
S_m &= \{|m+2n\rangle, n = 0, 1, 2, \ldots\} \\
S_{m+1} &= \{|m+2n+1\rangle, n = 0, 1, 2, \ldots\}.
\end{aligned} \tag{11}$$

S_0 and S_1 are both finite-dimensional and it is not possible to find operators $G_0^\dagger$ and $G_1^\dagger$ such that $[F, G_0^\dagger] = 1$ and $[F, G_1^\dagger] = 1$, respectively, hold in each sector. Since S_m and S_{m+1} are infinite-dimensional, the approach in [19] is applicable. (At the same time, it should be noticed that the two 'vacua' $|m\rangle$ and $|m+1\rangle$ originate directly from the zeros of the function $f(n, m)$ but not immediately from the annihilation action of a^2 in F; in this case the approach in [19] still applies. See reference 14 of [19].) Let $\mathcal{S}_m$ and $\mathcal{S}_{m+1}$ denote the spaces spanned by the sets of states S_m and S_{m+1}, and introduce two operators $G_m^\dagger$ and $G_{m+1}^\dagger$ by

$$G_k^\dagger = \frac{1}{2}F^\dagger \frac{1}{FF^\dagger}(N+2-k) \qquad k = m, m+1 \tag{12}$$

then

$$[F, G_k^\dagger]|\psi\rangle_k = |\psi\rangle_k \qquad \forall|\psi\rangle_k \in \mathcal{S}_k. \tag{13}$$

Substituting the explicit expression of $f(N, m)$ in equation (9) into equation (12), and taking into account the fact that in the sector S_k

$$N + 2 - k = 2\left[\frac{N-m+2}{2}\right] \tag{14}$$

we get

$$G_k^\dagger = \frac{a^{\dagger 2}}{2}. \tag{15}$$

Thus, apart from the normalization constants, the eigenstates of F are

$$|\alpha\rangle_m \sim \exp\left(\frac{\alpha}{2}a^{\dagger 2}\right)|m\rangle \qquad |\alpha\rangle_{m+1} \sim \exp\left(\frac{\alpha}{2}a^{\dagger 2}\right)|m+1\rangle. \tag{16}$$

It turns out that $|\alpha\rangle_m$ and $|\alpha\rangle_{m+1}$ are, respectively, the exponential forms of the PASV and the PASOPS.

4. Eigenstates of $f(N, -m)a^2$

Now that $f(n, -m)$ ($m \geqslant 0$) is also a well-defined function

$$f(n, -m) = \frac{2\left[\frac{n+m+2}{2}\right]}{(n+1)(n+2)} \tag{17}$$

we are naturally interested in what the eigenstates of $f(N, -m)a^2$ are. Let F' denote $f(N, -m)a^2$. It is easily seen that the states which are annihilated by F' are $|0\rangle$ and $|1\rangle$.

(Note that the function $f(n,-m)$ has no zeros at positive integer values of n and so F' has no 'additional' vacua such as $|m\rangle$ and $|m+1\rangle$ being F's vacua.) The sector S'_0 built out of $|0\rangle$ by repeated applications of $F'^\dagger$ is $S'_0 = \{|2n\rangle, n = 0, 1, \ldots\}$, while the sector S'_1 generated by consecutively applying $F'^\dagger$ on $|1\rangle$ is $S'_1 = \{|2n+1\rangle, n = 0, 1, \ldots\}$. S'_0 and S'_1 are both infinite-dimensional and thus the method in [19] applies. Let us introduce two operators $G'^\dagger_0$ and $G'^\dagger_1$ by

$$G'^\dagger_i = \frac{1}{2} F'^\dagger \frac{1}{F'F'^\dagger}(N+2-i) \qquad i = 0, 1 \tag{18}$$

that is,

$$G'^\dagger_0 = \frac{a^{\dagger 2}}{2} \frac{1}{(N+1)f(N,-m)} \qquad G'^\dagger_1 = \frac{a^{\dagger 2}}{2} \frac{1}{(N+2)f(N,-m)} \tag{19}$$

so that

$$[F', G'^\dagger_i] = 1 \qquad i = 0, 1. \tag{20}$$

Consequently the two eigenstates of F' are $e^{\alpha G'^\dagger_i}|i\rangle$ $(i = 0, 1)$. Inserting the expression of $f(N,-m)$ in equation (17) into (19) we have

$$G'^\dagger_0 = a^{\dagger 2} \frac{N+2}{4[\frac{N+m+2}{2}]} \equiv a^{\dagger 2} \mathcal{G}_0(N) \tag{21}$$

$$G'^\dagger_1 = a^{\dagger 2} \frac{N+1}{4[\frac{N+m+2}{2}]} \equiv a^{\dagger 2} \mathcal{G}_1(N). \tag{22}$$

Then, with the use of the relation

$$(G'^\dagger_0)^n = \begin{cases} 1 & n = 0 \\ a^{\dagger 2n} \mathcal{G}_0(N)\mathcal{G}_0(N+2)\ldots\mathcal{G}_0(N+2n-2) & n \geqslant 1 \end{cases} \tag{23}$$

and from the definition of $\mathcal{G}_0(N)$ in equation (21) we calculate

$$\begin{aligned} e^{\alpha G'^\dagger_0}|0\rangle &= \sum_{n=0}^{\infty} \frac{(\alpha G'^\dagger_0)^n}{n!}|0\rangle \\ &= |0\rangle + \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} a^{\dagger 2n} \mathcal{G}_0(N)\mathcal{G}_0(N+2)\ldots\mathcal{G}_0(N+2n-2)|0\rangle \\ &= |0\rangle + \sum_{n=1}^{\infty} \frac{\alpha^n \sqrt{(2n)!}}{n!} \mathcal{G}_0(0)\mathcal{G}_0(2)\ldots\mathcal{G}_0(2n-2)|2n\rangle \\ &= \sum_{n=0}^{\infty} \frac{\alpha^n \sqrt{(2n)!}}{\Gamma_m(n)}|2n\rangle \end{aligned} \tag{24}$$

wherein $\Gamma_m(n)$ is introduced by

$$\Gamma_m(n) \equiv \begin{cases} \prod_{i=1}^{n} \left(2\left[\frac{m+2i}{2}\right]\right) & n = 1, 2, \ldots \\ 1 & n = 0. \end{cases} \tag{25}$$

Similarly, one can derive

$$e^{\alpha G'^\dagger_1}|1\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n \sqrt{(2n+1)!}}{\Gamma_{m+1}(n)}|2n+1\rangle. \tag{26}$$

Thus, the normalized eigenstates of $f(N, -m)a^2$ ($m \geqslant 0$) are

$$|\alpha, -m\rangle_e = \mathcal{N}_e \sum_{n=0}^{\infty} \frac{\alpha^n \sqrt{(2n)!}}{\Gamma_m(n)} |2n\rangle \qquad |\alpha, -m\rangle_o = \mathcal{N}_o \sum_{n=0}^{\infty} \frac{\alpha^n \sqrt{(2n+1)!}}{\Gamma_{m+1}(n)} |2n+1\rangle \tag{27}$$

where $\mathcal{N}_e$ and $\mathcal{N}_o$ are normalization constants equal to

$$\mathcal{N}_e = \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}(2n)!}{(\Gamma_m(n))^2} \right)^{-1/2} \qquad \mathcal{N}_o = \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}(2n+1)!}{(\Gamma_{m+1}(n))^2} \right)^{-1/2}. \tag{28}$$

The subscripts e and o denote even and odd states, respectively. It follows from the above results that for an even m, $|\alpha, -m\rangle_e$ and $|\alpha, -(m+1)\rangle_e$ are identical; $|\alpha, -m\rangle_o$ and $|\alpha, -(m-1)\rangle_o$ are also the same. When $m = 0$, $|\alpha, -m\rangle_e$ reduces to the squeezed vacuum and $|\alpha, -m\rangle_o$ to the squeezed one-photon state. On the other hand, when $\alpha \to 0$, $|\alpha, -m\rangle_e$ degenerates to the vacuum state and $|\alpha, -m\rangle_o$ to the one-photon state. Put another way, $|\alpha, -m\rangle_e$ is *intermediate* between the squeezed vacuum and the vacuum state and $|\alpha, -m\rangle_o$ between the squeezed one-photon state and the one-photon state. It should be mentioned that the PASV is intermediate between the squeezed vacuum and the number state $|m\rangle$, and the PASOPS is between the squeezed one-photon state and the number state $|m+1\rangle$.

5. Nonclassical properties of $|\alpha, -m\rangle_e$ and $|\alpha, -m\rangle_o$

5.1. Photon statistics

A field is antibunched if its second-order correlation function $g^{(2)}(0) < 1$ [21], namely,

$$g^{(2)}(0) = \frac{\langle N^2 \rangle - \langle N \rangle}{\langle N \rangle^2} < 1. \tag{29}$$

For $|\alpha, -m\rangle_e$ and $|\alpha, -m\rangle_o$, the averages $\langle N^l \rangle$ ($l = 1, 2, \ldots$) are given by the following expressions:

$${}_e\langle \alpha, -m|N^l|\alpha, -m\rangle_e = \mathcal{N}_e^2 \sum_{n=0}^{\infty} \frac{(2n)^l (2n)! |\alpha|^{2n}}{(\Gamma_m(n))^2} \tag{30}$$

$${}_o\langle \alpha, -m|N^l|\alpha, -m\rangle_o = \mathcal{N}_o^2 \sum_{n=0}^{\infty} \frac{(2n+1)^l (2n+1)! |\alpha|^{2n}}{[\Gamma_{m+1}(n)]^2}. \tag{31}$$

In our numerical study, we find $g^{(2)}(0)$ for $|\alpha, -m\rangle_e$ is definitely greater than unity irrespective of the values of m, that is, the field in an even state $|\alpha, -m\rangle_e$ is always bunched. In contrast, the odd states $|\alpha, -m\rangle_o$ strongly exhibit the antibunching effect for any value of m. In figure 1 we plot $g^{(2)}(0)$ for $|\alpha, -m\rangle_o$, denoted by $g_o^{(2)}(0)$, against the parameter r for different values of m (recall that r is related to α through equation (7)). We find that $g_o^{(2)}(0)$ is a monotonically increasing function of r, however, when r is very large, $g_o^{(2)}(0)$ is almost independent of r. This is reasonable since when $r \to \infty$, $|\alpha| = \tanh r \to 1$. As a result, the maximum antibunching is achieved in the one-photon state ($r \to 0$). At the same time, it is clear that $g_o^{(2)}(0)$ decreases with increasing m. When $m \leqslant 4$, there is an interval of r within which $|\alpha, -m\rangle_o$ exhibits a bunching effect. When $m > 4$, the antibunching behaviour persists for the whole interval of r.

5.2. Quadrature squeezing

The quadrature operators of the single-mode field are defined as

$$X = \frac{1}{\sqrt{2}}(a + a^\dagger) \qquad P = \frac{1}{\sqrt{2}\mathrm{i}}(a - a^\dagger). \tag{32}$$

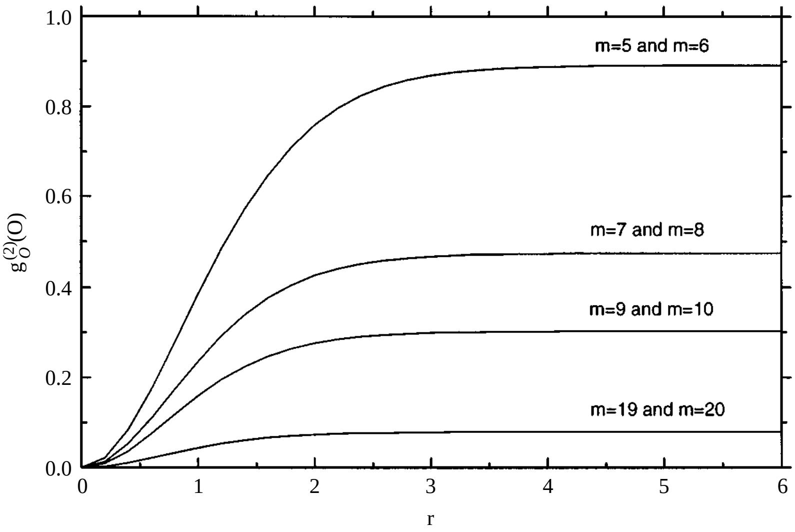

Figure 1. The second-order correlation function $g^{(2)}(0)$ of the odd states $|\alpha, -m\rangle_o$ as a function of the parameter r for two different values of m, r is related to α through $\alpha = 3D\mathrm{e}^{\mathrm{i}\theta}\tanh r \cdot g^{(2)}(0)$ is independent of θ.

They satisfy the commutation relation $[X, P] = i$ and consequently their variances $(\Delta X)^2 = \langle X^2\rangle - \langle X\rangle^2$, $(\Delta P)^2 = \langle P^2\rangle - \langle P\rangle^2$ obey the Heisenberg uncertainty relation

$$(\Delta X)^2(\Delta P)^2 \geqslant \tfrac{1}{4}. \tag{33}$$

The field is said to be squeezed in the X (P) quadrature if $(\Delta X)^2 < \frac{1}{2}$ ($(\Delta P)^2 < \frac{1}{2}$). For the sake of convenience, we define the squeezing indices as

$$S_X = 2(\Delta X)^2 - 1 \qquad S_P = 2(\Delta P)^2 - 1. \tag{34}$$

When $S_X < 0$ ($S_P < 0$), the field is squeezed in the X (P) quadrature. Note that for both states $|\alpha, -m\rangle_e$ and $|\alpha, -m\rangle_o$ we always have $\langle a\rangle = \langle a^\dagger\rangle = 0$ so that S_X and S_P are given by

$$S_X = \langle a^2 + a^{\dagger 2}\rangle + 2\langle N\rangle \qquad S_P = -\langle a^2 + a^{\dagger 2}\rangle + 2\langle N\rangle. \tag{35}$$

Using the definitions of $|\alpha, -m\rangle_e$ and $|\alpha, -m\rangle_o$, we obtain the following expressions:

$${}_e\langle\alpha, -m|a^2|\alpha, -m\rangle_e = \mathcal{N}_e^2 \sum_{n=0}^{\infty} \frac{(2n+2)!|\alpha|^{2n}\alpha}{\Gamma_m(n)\Gamma_m(n+1)} \tag{36}$$

$${}_o\langle\alpha, -m|a^2|\alpha, -m\rangle_o = \mathcal{N}_o^2 \sum_{n=0}^{\infty} \frac{(2n+3)!|\alpha|^{2n}\alpha}{\Gamma_{m+1}(n)\Gamma_{m+1}(n+1)}. \tag{37}$$

It is convenient to make numerical evaluations of the expressions above. The results show that there is no squeezing at all in the odd states $|\alpha, -m\rangle_o$, while the even states $|\alpha, -m\rangle_e$ could exhibit squeezing for any values of m and r, as we can appreciate in figure 2, in which S_X (denoted by S_{Xe} in the figure) is plotted against the parameter r for different values of m. In figure 2 we have chosen $\theta = \pi$ because in this case the maximum squeezing in the X quadrature is achieved. One can see that in this case X-squeezing always occurs irrespective of the values of m and r. The depth of squeezing is very sensitive to the values of m: as m decreases, the X-squeezing becomes more effective. This means that decreasing the degree of excitation can enhance the squeezing in X quadrature. It is interesting to note that the

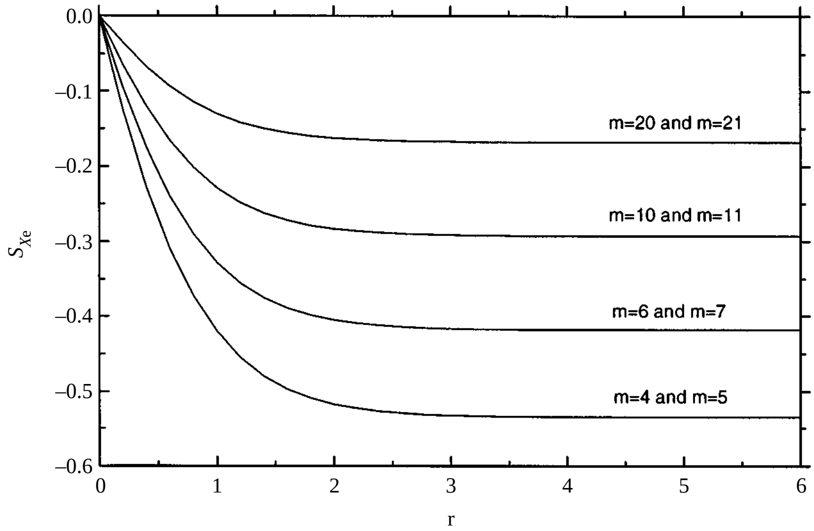

Figure 2. Squeezing index S_X of the even states $|\alpha, -m\rangle_e$ as a function of the parameter r for different values of m, r is related to $\alpha = 3De^{i\theta}\tanh r \cdot \theta$ is taken as π.

nonclassical behaviours of our new even and odd states are very similar to those of the even and odd coherent states introduced by Hillery [22]. In [23] Xia and Guo showed that the even coherent states can exhibit squeezing but not antibunching, however, the odd coherent states always exhibit antibunching but never squeezing.

6. Conclusion

In this paper we have shown that the PASV and PASOPS are even and odd NLCSs, respectively, that is, they are both the eigenstates of $\frac{2[\frac{N-m+2}{2}]}{(N+1)(N+2)}a^2$ ($m \geqslant 0$). On the basis of this fact, they can be equivalently cast into exponential-operator forms. Furthermore, we have constructed the eigenstates of $\frac{2[\frac{N+m+2}{2}]}{(N+1)(N+2)}a^2$. It turns out that the new states are highly nonclassical.

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